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Synthesis of Taylor and Bayliss Patterns for Linear Antenna Arrays

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The history of synthesis techniques for designing linear antenna arrays with low sidelobe patterns is reviewed briefly, and the limitations that are encountered with very low sidelobes and/or small arrays are pointed out. Taylor's continuous aperture synthesis procedure is outlined, and a technique for transforming it for application to a discrete array is described. Discrete-array design equations for Taylor and Bayliss synthesis procedures are given. A set of programs for use on a programmable calculator are presented.

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CONTENTS

SYNTHESIS OF TAYLOR AND BAYLISS PATTERNS FOR LINEAR ANTENNA ARRAYS

INTRODUCTION

The requirement for low sidelobes from array-type antennas is a long-standing one. The contributions to this theory extend from Dolph's utilization of Chebyshev polynomials, through Taylor's papers on linear and circular apertures, Bayliss's extension to difference-type patterns, and finally to recently developed techniques which provide arbitrary pattern control for linear arrays [1-8].

The purpose of this report is to examine some of the more recent applications of these synthesis techniques in light of their limitations and also the computational capabilities which are now available. For example, at the time Taylor published his synthesis procedure, engineers had only slide rules, mathematical tables, and mechanical desk calculators to generate the distribution functions. The computational capability available to today's engineer is vastly different, and we will show how Taylor's and Bayliss's procedures can be modified to give better results.

A more careful look at the synthesis procedures previously mentoned is presented in Table 1.

Dolph's synthesis is precise and gives minimum beamwidth for given sidelobe levels, but these constant amplitude sidelobes are not desirable for larger arrays because it is possible to radiate most of the energy into the sidelobes. Taylor solved this problem by allowing the far-out sidelobes to fall off as dictated by an amplitude discontinuity at the ends of the aperture. Taylor, and later Bayliss, synthesized continuous distributions and sampled these to obtain array excitations.

Table 1 — Synthesis Procedures for Linear Array Apertures

Procedure/ Date	Continuous or Discrete	Limitations
Dolph/47	Discrete	Poor results for large arrays
Taylor/52	Continuous	Inexact for low sidelobes, small arrays
Bayliss/68	Continuous	Inexact for low sidelobes, small arrays
Hyneman/68	Continuous	Inexact for low sidelobes, small arrays—iterative
Stutzman/72	Continuous	Inexact for low sidelobes, small arrays—iterative
Elliott/76	Continuous	Inexact for low sidelobes, small arrays—iterative
Elliott/77	Discrete	Applies all continuous procedures to discrete arrays

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Some recent applications have called for lower sidelobes and smaller arrays, thereby pressing the limitations of the Taylor and Bayliss synthesis procedures. The problem of discretizing continuous aperture distributions has been treated [9-10]. The technique used in this report is different from those of Winter and of Elliott, but it is mathematically related to Elliot's technique.

REVIEW OF TAYLOR SYNTHESIS PROCEDURE

A brief review of the Taylor synthesis procedure is given here. The key to this procedure is the equal-sidelobe pattern function which is the continuous-aperture analog to the Chebyshev polynomial pattern for arrays:

$$E(u) = \cos \pi \sqrt{u^2 - A^2}, \tag{1}$$

where $u = \pi a \sin \theta / \lambda$, a is the length of the aperture and θ is the angle measured relative to the normal to the array. This function has a maximum value of $\cosh \pi A$ at $u = 0$ and unit sidelobes extending to $u = \pm \infty$. Taylor showed that the pattern of Eq. (1) is not physically realizable from a continuous aperture distribution, just as the Dolph array excitation becomes increasingly impractical in the limit of large arrays. His brilliant solution to this problem was:

1. For all zeros of the synthesized pattern functions, which we will call $E_s(u)$, from the nth from the origin to ∞, the locations will be the same as those from a uniformly illuminated aperture of the same size. That is,

$$E_s(u) = 0 \text{ for } u = n \text{ for } n \geqslant \bar{n}.$$

2. For the first $\bar{n} - 1$ zeros, their locations will be determined by the zeros of $E(u)$, scaled so that the nth zero is located at $u = \bar{n}$.

The aperture distribution is determined by performing a Woodward synthesis of $E_s(u)$. That is, we define a set of functions of the form

$$F_n(u) = \sin (u - n)\pi/(u - n)\pi,$$

and then construct $E_s(u)$ from the $F_n(u)$

$$E_s(u) = \sum_{n=-\infty}^{\infty} E_s(n) F_n(u). \tag{2}$$

Since we have defined $E_s(n) = 0$ for $n \geqslant \bar{n}$, Eq. (3) becomes

$$E_s(u) = \sum_{n=-\bar{n}+1}^{\bar{n}-1} E_s(n) F_n(u). \tag{3}$$

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Fourier transformation of Eq. (3) yields the aperture distribution:

$$A(x) = \int_{-\infty}^{\infty} E_s(u)\, e^{j2xu\pi/a} du$$

$$= \int_{-\infty}^{\infty} \sum_{n=-\bar{n}+1}^{\bar{n}-1} E_s(n) F_n(u) e^{j2xu\pi/a} du. \tag{4}$$

That is, $A(x)$ is a weighted sum of integrals of the form,

$$\int_{-\infty}^{\infty} \frac{\sin (u - n)\pi}{(u - n)\pi} e^{j2xu\pi/a} du.$$

Letting $u' = u - n$ results in

$$e^{j2n\pi x/a} \int_{-\infty}^{\infty} \frac{\sin u'\pi}{u'\pi} e^{j2xu'\pi/a} du'.$$

Since the imaginary part of the integrand is odd, this becomes

$$e^{j2n\pi x/a} \int_{-\infty}^{\infty} \frac{\sin u'\pi \cos 2xu'\pi/a \, du'}{u'\pi}$$

$$= e^{j2n\pi x/a} \int_{-\infty}^{\infty} \frac{1}{2} \left[\frac{\sin u'\pi(1 - 2x/a) + \sin u'\pi(1 + 2x/a)}{u'\pi} \right] du'. \tag{5}$$

A standard definite integral is

$$\int_{-\infty}^{\infty} \frac{\sin bzdz}{z} \begin{array}{l} = \pi \text{ for } b > 0 \\ = 0 \text{ for } b = 0 \\ = -\pi \text{ for } b < 0 \end{array}$$

Application of this integral to Eq. (5) and thence to Eq. (4) yields

$$A(x) = \sum_{n=-\bar{n}+1}^{\bar{n}-1} E_s(n) e^{j2\pi nx/a}$$

$$= E_s(0) + 2 \sum_{n=1}^{\bar{n}-1} E_s(n) \cos 2\pi nx/a \text{ for } |x| \leqslant a/2 \tag{6}$$

$$= 0 \qquad \text{for } |x| > a/2.$$

The continuous aperture distribution given by Eq. (6) is sampled to give the element excitation values for a discrete array. This last step is approximate, and the pattern function of the array is obviously different from $E_s(u)$. This approximation is acceptable provided that the number of elements in the array is much greater than $\bar{n}$ and the sidelobe level is not extremely low. Figure 1 is an example of a case in which the synthesis procedure gives an unsatisfactory result. For a sidelobe level of 50 dB below mainbeam and $\bar{n} = 8$, a 30-element array has the computed pattern function shown. The near-in sidelobes are unduly low, whereas the first eight sidelobes should be about the same level.

ARRAY PATTERN FUNCTIONS IN TERMS OF ZEROS

Elliott used a synthesis technique which relates the discrete array distribution directly with the array pattern [9]. We also use this relationship, and our procedure achieves identical results with those of Elliott. However, the actual computations are different, and it is desirable to compare the techniques.

Elliott expresses the pattern function as a polynomial in w, where $w = e^{j(2\pi s/\lambda)\sin\theta}$. The zeros of this polynomial are given by w_n, which are normally located on the unit circle. Once he has the w_n properly adjusted, he completes the synthesis by multiplying out the product expression, $\Pi(w - w_n)$, into the polynomial. The coefficients of the polynomial are the excitations of the array elements.

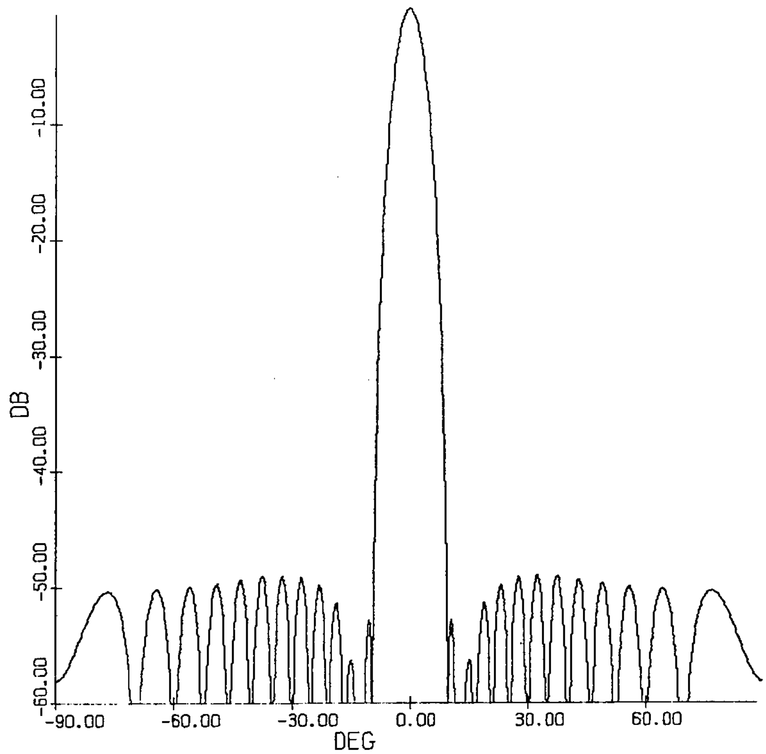

Fig. 1 — Conventional Taylor synthesis, $N = 30$, $\bar{n} = 8$, 50-dB sidelobes

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Our procedure also uses the pattern function zeros in a product expression. Since the patterns are symmetric, our expression can be of the form, $\Pi(\cos z - \cos z_n)$, where $z = (2\pi s/\lambda) \sin \theta$. We cannot multiply this product expression out to obtain the coefficients directly since we require terms of the form cos nz rather than $\cos^n z$. Rather, we carry out a synthesis exactly analogous to that used by Taylor. Uniformly spaced pattern function samples are found by using the product expression. These pattern samples are used in a Fourier series to find the array illumination.

The procedure relies on the equivalent location of pattern function zeros for the line source and for the discrete array. Whereas the zeros for the pattern of a uniform line source distribution are located at $u = n$, the analogous relationship for a discrete array is $z = n\pi/N$, where $z = 2\pi s \sin \theta/\lambda$, where s is element spacing and N is the number of elements in the uniformly excited array.

The transformation of Taylor's procedure is easily seen to consist of locating the zeros in step 1 above at $z = n\pi/N$ for $n \geqslant \bar{n}$ and then scaling the first $\bar{n}$ zeros of Eq. (1) so that the $\bar{n}$th zero is located at $z = \bar{n}\pi/N$.

Appendix A lists the resulting equations for Taylor arrays of both even and odd N, and Appendix B lists the equations for Bayliss arrays (yielding monopulse difference patterns) of both even and odd N. Figure 2 is an example of a Taylor array pattern with sidelobe levels of 50 dB with $\bar{n} = 8$ and $N = 30$. These equations can be straightforwardly programmed for automatic processing by a digital computer. Many programmable calculators now have sufficient memory to implement these programs. Appendix C lists programs for carrying out the synthesis and evaluating the pattern functions with an HP-41C programmable calculator.

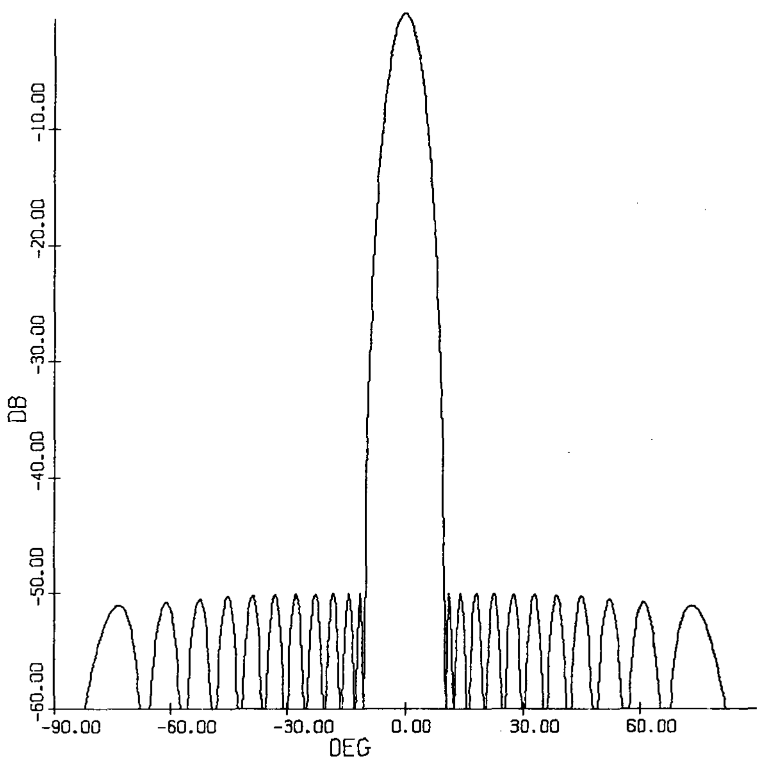

Fig. 2 — Discretized Taylor synthesis, $N = 30, \bar{n} = 8$, 50-dB sidelobes

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Appendix A

DESIGN EQUATIONS FOR LINEAR ARRAYS WITH TAYLOR-TYPE PATTERNS

These equations will determine the aperture illumination coefficients for a linear array of N elements to produce a Taylor-type pattern function with $\bar{n}$ sidelobes on each side of the main beam at a level of L dB.

This design procedure involves three steps. The first $\bar{n} - 1$ zeros of the pattern are determined. Then the appropriate pattern function samples are determined. Finally, the array element illumination coefficients are determined by a harmonic analysis of the pattern function samples.

A particular advantage of this synthesis is that the knowledge of all of the pattern function zeros allows the computation of the pattern function as a product rather than as a polynomial. The product computation involves only one trigonometric function evaluation for each pattern function value. All other constants need to be evaluated only once for each array.

The pattern function zeros are given by

$$z_n = \frac{2\pi\bar{n}\sqrt{A^2 + (n - 1/2)^2}}{N\sqrt{A^2 + (\bar{n} - 1/2)^2}} \qquad \text{for } n = 1 \text{ to } \bar{n} - 1 \tag{A1a}$$

$$= \frac{2\pi n}{N} \qquad \text{for } n = \bar{n} \text{ to } M\,, \tag{A1b}$$

where

$$M = \mathrm{int}\left(\frac{N - 1}{2}\right)$$

and A is given by

$$A = \frac{1}{\pi}\cosh^{-1}\left[10^{(L/20)}\right] \tag{A2a}$$

$$\approx (L + 6.02)/27.29\,, \tag{A2b}$$

where L is the sidelobe level (positive) in dB. Equation (A2b) is an excellent approximation, especially for large L.

The pattern function is given by

$$E(z) = \cos\frac{z}{2} \prod_{n=1}^{M} \left(\frac{\cos z - \cos z_n}{1 - \cos z_n} \right) \qquad N \text{ even}$$

$$= \prod_{n=1}^{M} \left(\frac{\cos z - \cos z_n}{1 - \cos z_n} \right) \qquad N \text{ odd} \tag{A3}$$

The pattern samples to be used to find the array element illumination coefficients are given by

$$a_m = E\left(\frac{2\pi m}{N}\right) \qquad \text{for } m = 1 \text{ to } \bar{n} - 1 \,. \tag{A4}$$

The element excitation coefficients are given by

$$e_p = 1 + 2 \sum_{m=1}^{\bar{n}-1} a_m \cos\frac{m(2p-1)\pi}{N} \qquad N \text{ even, } p = 1 \text{ to } M + 1$$

$$= 1 + 2 \sum_{m=1}^{\bar{n}-1} a_m \cos\frac{2mp\pi}{N} \qquad N \text{ odd, } p = 0 \text{ to } M \,, \tag{A5}$$

where p is an index or element number starting at the center and moving to either end of the array.

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Appendix B

DESIGN EQUATIONS FOR LINEAR ARRAYS WITH BAYLISS-TYPE DIFFERENCE PATTERNS

Appendix A gave the design equations for linear arrays with Taylor-type patterns, which produce a main beam with slightly larger beamwidth than that of the Dolph synthesis but in general with higher gain. In some applications; such as monopulse, we might require a difference pattern. Bayliss presented a synthesis procedure for difference patterns, analogous to that of Taylor. In this appendix we adapt the Bayliss procedure to discrete arrays.

As in the case of the Taylor synthesis, the application of discrete arrays involves three steps. The first $\overline{n} - 1$ off-axis zeros of the pattern are determined. Then the appropriate pattern function samples are determined. Finally the array element illumination coefficients are determined by a harmonic analysis of the pattern function samples.

The pattern function zeros are given by

$$z_n = \frac{2\pi q_n\left(\overline{n} + \frac{1}{2}\right)}{N\sqrt{A^2 + \overline{n}^2}} \qquad \text{for } n = 1, 2, 3, 4 \tag{B1a}$$

$$= \frac{2\pi\left(\overline{n} + \frac{1}{2}\right)\sqrt{A^2 + n^2}}{N\sqrt{A^2 + \overline{n}^2}} \qquad \text{for } n = 5 \text{ to } \overline{n} - 1 \tag{B1b}$$

$$= \frac{2\pi\left(n + \frac{1}{2}\right)}{N} \qquad \text{for } n = \overline{n} \text{ to } M \tag{B1c}$$

where

$$M = \text{int}\left(\frac{N-2}{2}\right).$$

In this case it is necessary to find both A and q_n from graphs in Bayliss's paper [4]. For 50 dB sidelobes, $A = 2.42$, $q_1 = 2.78$, $q_2 = 3.18$, $q_3 = 3.85$, and $q_4 = 4.65$.

The pattern function is given by

$$E(z) = \sin\frac{z}{2}\prod_{n=1}^{M}\left[\cos z - \cos z_n\right] \Big/ \sin\frac{z_1}{4}\prod_{n=1}^{M}\left[\cos\frac{z_1}{2} - \cos z_n\right] \qquad N \text{ even} \tag{B2}$$

$$= \sin z\prod_{n=1}^{M}\left[\cos z - \cos z_n\right] \Big/ \sin\frac{z_1}{2}\prod_{n=1}^{M}\left[\cos\frac{z_1}{2} - \cos z_n\right] \qquad N \text{ odd}$$

$E(z)$ is normalized to unity at $z = z_1/3$, which is near the pattern maximum. If a more precise pattern maximum is desired, a better multiplying constant can easily be found.

The pattern samples to be used to find the array element illumination coefficients are given by

$$b_m = E\left(\frac{\pi}{N}(2m - 1)\right) \qquad \text{for } m = 1 \text{ to } \bar{n}\,. \tag{B3}$$

The element excitation coefficients are given by

$$\begin{aligned} e_p &= 2\sum_{m=1}^{\bar{n}} b_m \sin\frac{\pi(2m-1)(2p-1)}{2N} && \text{for } N \text{ even, } p = 1 \text{ to } M + 1 \\ &= 2\sum_{m=1}^{\bar{n}} b_m \sin\frac{\pi(2m-1)p}{N} && \text{for } N \text{ odd, } p = 1 \text{ to } M + 1 \end{aligned} \tag{B4}$$

where p is an index of the element number starting with zero at the center of the array. For N odd, the center element of the array always has zero excitation. The excitations on one side of the array are the negative of those on the other side.

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Appendix C

PROGRAMS FOR THE HP-41C CALCULATOR

This appendix presents programs for the HP-41C calculator for the design equations of Appendices A and B. The software consists of four programs, SUM, DIF, IN, and SL. "SUM" contains the equations for synthesizing Taylor-type sum patterns; "DIF" contains equations for Bayliss-type difference patterns; "IN" contains subroutines that are used by both programs; and "SL" is a routine for calculating the peaks of the sidelobes of the synthesized array. The number of registers used by the programs and the number of card sides required for storage are:

Program	Registers	Card Sides
SUM	30	2
DIF	42	3
IN	39	3
SL	19	2
	130	10 (5 cards) .

It is possible to synthesize aperture distributions using either SUM and IN or DIF and IN. These programs require at least one additional memory module. Furthermore, the programs use nine registers for variables, indices, and constants. Table C1 correlates the number of registers available for synthesis parameters with the number of additional memory modules in use. The available registers are used for the pattern samples a_m and b_m and for the pattern function zeros (cosines) z_p. The number of these registers is $\overline{n} + M$. Therefore, the size of array that can be synthesized for any given configuration of Table C1 depends on $\overline{n}$. For a 50-dB sidelobe requirement, $\overline{n}$ will be about 8. Roughly speaking, an array of 55 to 65 elements for difference and 80 to 90 for sum can be synthesized using one memory module by trading programs in and out of the machine, and an array of 90 to 100 elements can be synthesized with all programs loaded using two modules. The maximum array size that can be handled using three modules is 310 to 320 for difference and about 340 for sum. It appears that one or two memory modules should suffice for most requirements.

Table C1 — Registers Available after Loading Indicated Program Complements

Program Complement	Number of Memory Modules		
	1	2	3
SUM + IN	48	112	176
DIF + IN	35	99	163
SUM + DIF + IN	11	71	135
SUM + DIF + IN + SL	—	54	118

The procedure for running the programs is:

1. Allocate memory by XEQ SIZE $(9 + M + n)$.

2. Load the appropriate program complement.

3. Enter either XEQ SUM or XEQ DIF.

4. The display will prompt for N, L, and $NBAR$. N can be even or odd. L is sidelobe level in positive dB. DIF will also prompt for A, $Q1$, $Q2$, $Q3$, $Q4$.

5. After calculating z_n and loading $\cos z_n$ into registers starting with $(9 + \bar{n})$, the display will ask whether you want a listing of peak sidelobes (SL) or aperture distribution (EP). After the sidelobes or excitation coefficients are listed, the display will ask whether you want the other set of parameters calculated and listed.

The routine SL computes the sidelobe level relative to the main beam level by evaluating the pattern value at a point midway between pattern zeros. This computation is admittedly approximate because the pattern maximum is in general not exactly midway between zeros. The main beam pattern value is computed for $z = 0$. The difference pattern maximum is computed for $z = z_1/3$. This factor was found to be accurate for 50 dB sidelobes. The exact multiplying factor will be somewhat larger for higher sidelobes ($L < 50$), and it can be found quickly by obtaining z_1 and executing PA:

RCL $(9 + \bar{n})$	gives $\cos z_1$
ACOS	gives z_1
k	new multiplying factor, such as .4
*	
COS	
STO 02	
XEQ PA .	

Alternatively, k can be found from Fig. 4 of Bayliss[C1], which defines the beam maximum by p_o, where $k = p_o/\S_1$. ($\S_1$ corresponds to our z_1)

Once the desired value of k has been found, go to lines 110, 111 in DIF, and exchange k, * for 3,/. It is now necessary to reload the reference main beam pattern value into R08. This calculation starts at line 61 of SUM and 105 of DIF. Alternatively, you can simply rerun the program.

The pattern value, in voltage and normalized to mainbeam level, is found by keying in the value of z in degrees, then keying COS, STO 02, XEQ PA.

The registers used are:

00	N
01	M
02	A^2 and $\cos z_m$ for PA
03	n
04, 05	loop indices
06	multiplying constants

[C1] E. T. Bayliss, BSTJ, May-Jun 1968, pp. 623-650.

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07	accumulator for $E(z)$, e_p
08	main beam reference value
09 to $(8 + \bar{n})$	computed values of a_m, b_m
$(9 + \bar{n})$ to $(8 + \bar{n} + M)$	computed values of cos z_n.

Program IN contains the following subroutines:

IN	Asks for input data N, L, $NBAR$
ZN	Completes calculation and storage of cos z_n
BR	Asks for choice of sidelobes or aperture distribution and branches to EP or SL
PR	Prints element excitations e_p
PA	Computes pattern value for a_m, b_m, or SL routines
EP	Completes calculation of e_p.

The programs use flags 00 and 01 to indicate the following conditions:

Flag 00 is set for N even
clear for N odd

Flag 01 is set for DIF execution
clear for SUM execution.

The use of registers by program PA precludes the use of the plot subroutines resident in the printer.

Note that the sidelobes and pattern values obtained with these programs are all relative to the main beam level. No information concerning gain or aperture illumination efficiency is computed. The aperture distribution can be used to compute aperture efficiency or gain.

The programs and sample printouts are listed on the following pages.

```
01♦LBL "SUM"
 02 CF 01
 03 XEQ "IN"
 04 RCL 00
 05 1
 06 -
 07 2
 08 /
 09 INT
 10 STO 01
 11 360
 12 RCL 03
 13 *
 14 RCL 00
 15 /
 16 RCL 03
 17 .5
 18 -
 19 X↑2
 20 RCL 02
 21 +
 22 SQRT
 23 /
 24 STO 06
 25 RCL 03
 26 1
 27 -
 28 1 E-3
 29 *
 30 1
 31 +
 32 STO 04

33♦LBL 01
 34 RCL 04
 35 INT
 36 .5
 37 -
 38 X↑2
 39 RCL 02
 40 +
 41 SQRT
 42 XEQ "ZN"
 43 ISG 04
 44 GTO 01
 45 RCL 01
 46 1 E-3
 47 *
 48 RCL 03
 49 +
 50 STO 04
 51 360
 52 RCL 00
 53 /
 54 STO 06

55♦LBL 02
 56 RCL 04
 57 INT
 58 XEQ "ZN"
 59 ISG 04
 60 GTO 02
 61 1
 62 STO 02
 63 STO 08
 64 XEQ "PA"
 65 STO 08
 66 XEQ "BR"

67♦LBL "S"
 68 RCL 05
 69 1
 70 -
 71 1 E-3
 72 *
 73 1
 74 +
 75 STO 05
 76 360
 77 RCL 00
 78 /
 79 STO 06

80♦LBL 03
 81 RCL 06
 82 RCL 05
 83 INT
 84 *
 85 COS
 86 STO 02
 87 XEQ "PA"
 88 RCL 05
 89 8
 90 +
 91 X<>Y
 92 STO IND Y
 93 ISG 05
 94 GTO 03
 95 90
 96 RCL 00
 97 /
 98 STO 06
 99 1
100 1
101 FC? 00
102 0
103 FC? 00
104 0
105 RCL 01
106 +
107 1 E-3
108 *
109 +
110 STO 04

111♦LBL 04
112 1
113 RCL 03
114 1
115 -
116 1 E-3
117 *
118 +
119 STO 05
120 0
121 STO 07

122♦LBL 05
123 XEQ "EP"
124 ISG 05
125 GTO 05
126 2
127 RCL 07
128 *
129 1
130 +
131 XEQ "PR"
132 GTO 04
133 END


PRP "DIF"

01♦LBL "DIF"
 02 SF 01
 03 XEQ "IN"
 04 RCL 00
 05 2
 06 -
 07 2
 08 /
 09 INT
 10 STO 01
 11 "A="
 12 PROMPT
 13 FIX 2
 14 ARCL X
 15 PRA
 16 X↑2
 17 STO 02
 18 360
 19 RCL 00
 20 /
 21 RCL 02
 22 RCL 03
 23 X↑2
 24 +
 25 SQRT
 26 /
 27 RCL 03
 28 .5
 29 +
 30 *
 31 STO 06
 32 5
 33 RCL 03
 34 X<Y?
 35 GTO 07
 36 1.004
 37 STO 04
 38 GTO 06

39♦LBL 07
 40 1
 41 -
 42 1 E-3
 43 *
 44 +
 45 STO 04

46♦LBL 06
 47 RCL 04
 48 INT
 49 "ENT Q"
 50 FIX 6
 51 ARCL X
 52 PROMPT
 53 X<>Y
 54 "Q"
 55 ARCL X
 56 ACA
 57 "="
 58 ACA
 59 RDN
 60 FIX 2
 61 ACX
 62 PRBUF
 63 XEQ "ZN"
 64 ISG 04
 65 GTO 06
 66 5
 67 RCL 03
 68 1
 69 -
 70 X<Y?
 71 GTO 10
 72 1 E-3
 73 +
 74 +
 75 STO 04
```

```
76♦LBL 09
77 RCL 02
78 RCL 04
79 INT
80 X↑2
81 +
82 SQRT
83 XEQ "ZN"
84 ISG 04
85 GTO 09

86♦LBL 10
87 360
88 RCL 00
89 /
90 STO 06
91 RCL 03
92 RCL 01
93 1 E-3
94 *
95 +
96 STO 04

97♦LBL 11
98 RCL 04
99 INT
100 .5
101 +
102 XEQ "ZN"
103 ISG 04
104 GTO 11
105 RCL 03
106 9
107 +
108 RCL IND X
109 ACOS
110 3
111 /
112 COS
113 STO 02
114 1
115 STO 08
116 XEQ "PA"
117 STO 08
118 XEQ "BR"

119♦LBL "Q"
120 RCL 03
121 1 E-3
122 *
123 1
124 +
125 STO 05
126 180
127 RCL 00
128 /
129 STO 06

130♦LBL 14
131 RCL 05
132 INT
133 2
134 *
135 1
136 -
137 RCL 06
138 *
139 COS
140 STO 02
141 XEQ "PA"
142 RCL 05
143 8
144 +
145 X<>Y
146 STO IND Y
147 ISG 05
148 GTO 14
149 RCL 01
150 1
151 +
152 1 E-3
153 *
154 1
155 +
156 STO 04
157 90
158 RCL 00
159 /
160 STO 06

161♦LBL 15
162 1
163 RCL 03
164 1 E-3
165 *
166 +
167 STO 05
168 0
169 STO 07

170♦LBL 16
171 XEQ "EP"
172 ISG 05
173 GTO 16
174 2
175 RCL 07
176 *
177 XEQ "PR"
178 GTO 15
179 STOP
180 END

01♦LBL "IN"
02 FIX 0
03 CF 00
04 "N="
05 PROMPT
06 ARCL X
07 PRA
08 STO 00
09 2
10 /
11 ENTER↑
12 FRC
13 X=0?
14 SF 00
15 "L="
16 PROMPT
17 ARCL X
18 PRA
19 20
20 /
21 2
22 LOG
23 +
24 1
25 E↑X
26 LOG
27 PI
28 *
29 /
30 X↑2
31 STO 02
32 "NBAR="
33 PROMPT
34 ARCL X
35 PRA
36 STO 03
37 RTN

38♦LBL "ZN"
39 RCL 06
40 *
41 COS
42 RCL 03
43 8
44 +
45 RCL 04
46 +
47 X<>Y
48 STO IND
49 RTN

50♦LBL "BR"
51 "SL? 1/EP? 0"
52 PROMPT
53 X=0?
54 RTN
55 GTO "SL"

56♦LBL "PR"
57 "E"
58 RCL 04
59 INT
60 FIX 0
61 ARCL X
62 ACA
63 FIX 4
64 "="
65 ACA
66 RDN
67 ACX
68 PRBUF
69 ISG 04
70 RTN
71 ADV
72 "SL? 0"
73 PROMPT
74 X=0?
75 GTO "SL"
76 STOP

77♦LBL "PA"
78 RCL 01
79 1 E-3
80 *
81 1
82 +
83 STO 04
84 1
85 STO 07

86♦LBL 00
87 RCL 04
88 RCL 03
89 +
90 8
91 +
92 RCL 02
93 RCL IND Y
94 -
95 ST* 07
96 ISG 04
97 GTO 00
98 1
99 FS? 00
100 GTO 01
101 FC? 01
102 GTO 03
103 RCL 02
```


UNCLASSIFIED

```
104 ACOS
105 SIN
106 GTO 03

107+LBL 01
108 RCL 02
109 ACOS
110 2
111 /
112 FS? 01
113 GTO 02
114 COS
115 GTO 03

116+LBL 02
117 SIN

118+LBL 03
119 RCL 07
120 *
121 RCL 08
122 /
123 RTN

124+LBL "EP"
125 -1
126 FC? 00
127 0
128 RCL 04
129 INT
130 2
131 *
132 +
133 RCL 05
134 INT
135 2
136 *
137 FC? 01
138 GTO 03
139 1
140 -

141+LBL 03
142 *
143 RCL 06
144 *
145 FS? 01
146 GTO 04
147 COS
148 GTO 05

149+LBL 04
150 SIN

151+LBL 05
152 RCL 05
153 8
154 +
155 X<>Y
156 RCL IND Y
157 *
158 ST+ 07
159 RTN
160 END
```

```
PRP "SL"

01+LBL "SL"
02 "SL PEAKS, DB"
03 PRA
04 FIX 2
05 1
06 RCL 03
07 1
08 -
09 1 E-3
10 *
11 +
12 STO 05

13+LBL 00
14 RCL 03
15 8
16 +
17 RCL 05
18 +
19 RCL IND X
20 ACOS
21 X<>Y
22 1
23 +
24 X<>Y
25 RCL IND Y
26 ACOS
27 +
28 2
29 /
30 COS
31 XEQ 03
32 ISG 05
33 GTO 00
34 RCL 03
35 RCL 01
36 1 E-3
37 *
38 +
39 STO 05
40 180
41 RCL 00
42 /
43 STO 06

44+LBL 01
45 1
46 FS? 01
47 0
48 RCL 05
49 INT
50 2
51 *
52 +
53 RCL 06
54 *
55 COS
56 STO 02
57 XEQ 03
58 ISG 05
59 GTO 01
60 ADV
61 STOP

62+LBL 03
63 STO 02
64 XEQ "PA"
65 ABS
66 LOG
67 20
68 *
69 CHS
70 PRX
71 RTN
72 "EP? 0"
73 PROMPT
74 X≠0?
75 STOP
76 FS? 01
77 GTO "0"
78 GTO "S"
79 END
```

UNCLASSIFIED

```
N=20
L=50
NBAR=8
E1= 1.9452
E2= 1.8439
E3= 1.6549
E4= 1.4021
E5= 1.1166
E6= 0.8294
E7= 0.5684
E8= 0.3527
E9= 0.1903
E10= 0.0965

SL PEAKS, DB
          50.33    ***
          49.93    ***
          49.82    ***
          49.72    ***
          49.60    ***
          49.45    ***
          49.29    ***
          49.09    ***
          49.09    ***
```

```
N=21
L=50
NBAR=8
E0= 1.9579
E1= 1.9111
E2= 1.7766
E3= 1.5702
E4= 1.3156
E5= 1.0403
E6= 0.7766
E7= 0.5291
E8= 0.3306
E9= 0.1812
E10= 0.0957

SL PEAKS, DB
          50.35    ***
          49.95    ***
          49.86    ***
          49.77    ***
          49.67    ***
          49.56    ***
          49.43    ***
          49.26    ***
          49.29    ***
          49.31    ***
```

```
N=20
L=50
NBAR=8
A=2.42
Q1= 2.78
Q2= 3.18
Q3= 3.85
Q4= 4.65
E1= 0.4886
E2= 1.3673
E3= 1.9831
E4= 2.2448
E5= 2.1556
E6= 1.8001
E7= 1.3679
E8= 0.8180
E9= 0.4184
E10= 0.1751

SL PEAKS, DB
          49.54    ***
          48.41    ***
          47.91    ***
          46.89    ***
          47.09    ***
          46.90    ***
          46.55    ***
          46.56    ***
          45.64    ***
```

```
N=21
L=50
NBAR=8
A=2.42
Q1= 2.78
Q2= 3.18
Q3= 3.85
Q4= 4.65
E1= 0.9093
E2= 1.6548
E3= 2.1187
E4= 2.2516
E5= 2.0835
E6= 1.7013
E7= 1.2231
E8= 0.7650
E9= 0.3953
E10= 0.1726

SL PEAKS, DB
          49.56    ***
          48.44    ***
          47.94    ***
          46.93    ***
          47.15    ***
          46.98    ***
          46.64    ***
          46.65    ***
          45.93    ***
```